MULTIMEDIA UNIVERSITY

STUDENT ID NO

					4	2	5	7	9

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2017/2018

TMA1101 – CALCULUS

(All sections / Groups)

5 MARCH 2018
2:30 p.m. – 4:30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of six pages with **FIVE** questions.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the answer booklet provided.
4. **No calculators are allowed**.
5. **You are required to write proper steps.**

ANSWER ALL QUESTIONS.

QUESTION 1 [10 marks]

1 (a) Find the following limits.

[***You must show at least one intermediate step where*** $\lim_{x\to c}$ ***is still needed.***]

(i) $\lim_{x\to 5}\dfrac{2x^2-7x-15}{x-5}$

(ii) $\lim_{x\to\infty}\dfrac{x^2+\sin x}{3x^2+6}$

[2 marks]

(b) Given $f(x)=\begin{cases} x+3, & \text{if } x\le 3 \\ 2x, & \text{if } 3<x\le 4 \\ x^2-8, & \text{if } x>4 \end{cases}$

(i) Find $f(3)$.

(ii) Determine $\lim_{x\to 3^-} f(x)$ and $\lim_{x\to 3^+} f(x)$.

[***For this part, you must show at least one intermediate step where*** $\lim_{x\to 3^-}$ ***or*** $\lim_{x\to 3^+}$ ***is still needed.***]

(iii) Does $\lim_{x\to 3} f(x)$ exist? Give your reason. If it exists, state its value.

(iv) Is the function f continuous at 3? Give the reason for your answer.

[4.5 marks]

(c) (i) State the Intermediate Value Theorem (i.e., the full statement including the hypothesis and the conclusion).

(ii) Show that there is a root of the equation $x^5-5x^3+3=0$ in the interval $[-3,-2]$.

You must write proper steps to arrive at the conclusion; just writing some calculations would not be enough.

[3.5 marks]

QUESTION 2 [10 marks]

(a) Use the formal definition of derivative to find $f'(2)$ when $f(x) = (x+1)^2$.
You are reminded to write proper steps. [2.5 marks]

(b) Find $\frac{dy}{dx}$ with y as given.
[Use the product rule or the quotient rule for differentiation; show proper steps.]

(i) $y = e^x\sqrt{x}$

(ii) $y = \frac{x}{3+\sin x}$

[3 marks]

(c) The point (1, 2) lies on the curve $4y^2 + 3xy + 2x^3 = 24$.
Use implicit differentiation to obtain $\frac{dy}{dx}$ **in terms of x and y.**
Then find the equation of the tangent to the curve $4y^2 + 3xy + 2x^3 = 24$ at the point (1, 2).

[4.5 marks]

QUESTION 3 [10 marks]

(a) (i) Use $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$ and $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$ to find the values of A and B which make the equation $\sin 2\theta \sin 3\theta = A\cos 5\theta + B\cos\theta$ an identity.

(ii) Evaluate $\int_0^{\frac{\pi}{2}} \sin 2x \sin 3x\, dx$

[3.5 marks]

(b) (i) Determine the values of A and B in the following partial fraction decomposition.

$$\frac{x+10}{2x^2+5x-3} = \frac{A}{2x-1} + \frac{B}{x+3}$$

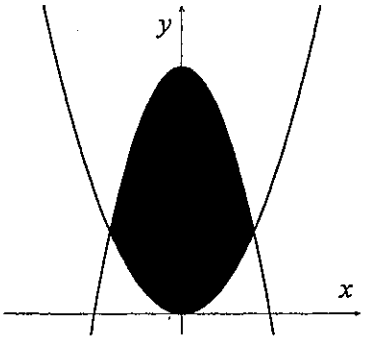
(ii) Integrate

$$\int \frac{x+10}{2x^2+5x-3} dx$$

[3 marks]

(c)

The figure shows a region bounded by two parabolas $y = x^2$ and $y = 12 - 2x^2$.



(i) Determine the x-coordinates of the points of intersection between the two parabolas.

(ii) Write down a definite integral that can be used to find the area of this region and proceed to find the area.

[3.5 marks]

QUESTION 4 [10 marks]

(a) Write down the condition on r for the convergence and divergence of the infinite **geometric series** $\sum_{n=1}^{\infty} ar^{n-1}$. $(a \neq 0)$

Then determine if the geometric series $\sum_{n=1}^{\infty} \frac{7^{k+1}}{5^{2k}}$ is convergent or divergent.

[2 marks]

(b) Use the **ratio test** to determine whether the following series is convergent.

$$\sum_{n=1}^{\infty} \frac{5^n}{n^2}$$

[2 marks]

(c) Find the **Maclaurin polynomial** of order 3 for $f(x) = \ln(1+x)$.

[3 marks]

(d) A periodic function $f(x)$ with period 2π is defined as

$$f(x) = \begin{cases} 1, & -\pi \leq x < -\frac{\pi}{2} \\ 0, & -\frac{\pi}{2} \leq x < \pi \end{cases}$$

(i) Sketch a graph of $f(x)$ in the interval $-2\pi < x < 2\pi$

(ii) The **Fourier series** of $f(x)$ has the form

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx).$$

Determine the value of b_3.

[3 marks]

QUESTION 5 [10 marks]

(a) Given $F(x,y) = y^2 + \cos 5x - e^{xy}$, find the partial derivatives $\frac{\partial F}{\partial x}$ and $\frac{\partial F}{\partial y}$.

[1 mark]

(b) Solve the first order **separable equation** $\frac{dy}{dx} - 2xy = x$

subject to the initial condition $y(2) = 0$.

You may leave your answer in implicit form. [2.5 marks]

(c) You are told that e^{-2x} is an integrating factor for the first order linear equation $\frac{dy}{dx} - 2y = 3e^{2x}$ subject to the initial condition $y(0) = 1$.

Solve the equation and give your solution in explicit form.

[3 marks]

(d) (i) Find the roots of the characteristics equation of the homogeneous differential equation $y'' - y' = 0$.

Then write down the general solution y_h of this homogeneous equation.

(ii) Find a particular solution of the second order differential equation $y'' - y' = e^{2x}$.

(iii) Hence, write down the general solution for the differential equation $y'' - y' = e^{2x}$

[3.5 marks]

End of Page